Number Systems

Knox Game Design

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Base 10 - Decimal

1 digit
0
1
2
3
4
5
6
7
8
9

2 digits
10
11
12
13
14
15
20
25
58
99

3 digits
100
101
102
110
153
200
256
777
876
999

4 digits
1000
1234
9999

Base 10 - Decimal

- $10^0 = 1$
- $10^1 = 10$
- $10^2 = 100$
- $10^3 = 1{,}000$
- $10^4 = 10{,}000$
- $10^5 = 100{,}000$
- $10^6 = 1{,}000{,}000$

Base 10 - Decimal

0	$0 * 10^0$
1	$1 * 10^0$
2	$2 * 10^0$
3	$3 * 10^0$
4	$4 * 10^0$
5	$5 * 10^0$
6	$6 * 10^0$
7	$7 * 10^0$
8	$8 * 10^0$
9	$9 * 10^0$

10	$(1 * 10^1) + (0 * 10^0)$
11	$(1 * 10^1) + (1 * 10^0)$
12	$(1 * 10^1) + (2 * 10^0)$
13	$(1 * 10^1) + (3 * 10^0)$
14	$(1 * 10^1) + (4 * 10^0)$
15	$(1 * 10^1) + (5 * 10^0)$
20	$(2 * 10^1) + (0 * 10^0)$
25	$(2 * 10^1) + (5 * 10^0)$
58	$(5 * 10^1) + (8 * 10^0)$
99	$(9 * 10^1) + (9 * 10^0)$

100	$(1 * 10^2) + (0 * 10^1) + (0 * 10^0)$
101	$(1 * 10^2) + (0 * 10^1) + (1 * 10^0)$
102	$(1 * 10^2) + (0 * 10^1) + (2 * 10^0)$
110	$(1 * 10^2) + (1 * 10^1) + (0 * 10^0)$
153	$(1 * 10^2) + (5 * 10^1) + (3 * 10^0)$
200	$(2 * 10^2) + (0 * 10^1) + (0 * 10^0)$
256	$(2 * 10^2) + (5 * 10^1) + (6 * 10^0)$
777	$(7 * 10^2) + (7 * 10^1) + (7 * 10^0)$
876	$(8 * 10^2) + (7 * 10^1) + (6 * 10^0)$
999	$(9 * 10^2) + (9 * 10^1) + (9 * 10^0)$

1000	$(1 * 10^3) + (0 * 10^2) + (0 * 10^1) + (0 * 10^0)$
1234	$(1 * 10^3) + (2 * 10^2) + (3 * 10^1) + (4 * 10^0)$
9999	$(9 * 10^3) + (9 * 10^2) + (9 * 10^1) + (9 * 10^0)$

KNOX GAME DESIGN

Base 2 - Binary

1 bit
0
1

2 bits
00
01
10
11

3 bits
000
001
010
011
100
101
110
111

4 bits
0000
0001
0010
1000
1001
1010
1011
1111

Base 2 - Binary

- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- $2^8 = 256$
- $2^9 = 512$
- $2^{10} = 1024$
- $2^{11} = 2048$
- $2^{12} = 4096$
- $2^{13} = 8192$

Binary to decimal

0	$0 * 2^0$	0
1	$1 * 2^0$	1

1 bit

00	$(0 * 2^1) + (0 * 2^0)$	0
01	$(0 * 2^1) + (1 * 2^0)$	1
10	$(1 * 2^1) + (0 * 2^0)$	2
11	$(1 * 2^1) + (1 * 2^0)$	3

2 bits

000	$(0 * 2^2) + (0 * 2^1) + (0 * 2^0)$	0
001	$(0 * 2^2) + (0 * 2^1) + (1 * 2^0)$	1
010	$(0 * 2^2) + (1 * 2^1) + (0 * 2^0)$	2
011	$(0 * 2^2) + (1 * 2^1) + (1 * 2^0)$	3
100	$(1 * 2^2) + (0 * 2^1) + (0 * 2^0)$	4
101	$(1 * 2^2) + (0 * 2^1) + (1 * 2^0)$	5
110	$(1 * 2^2) + (1 * 2^1) + (0 * 2^0)$	6
111	$(1 * 2^2) + (1 * 2^1) + (1 * 2^0)$	7

3 bits

0000	$(0 * 2^3) + (0 * 2^2) + (0 * 2^1) + (0 * 2^0)$	0
0001	$(0 * 2^3) + (0 * 2^2) + (0 * 2^1) + (1 * 2^0)$	1
0010	$(0 * 2^3) + (0 * 2^2) + (1 * 2^1) + (0 * 2^0)$	2
1000	$(1 * 2^3) + (0 * 2^2) + (0 * 2^1) + (0 * 2^0)$	8
1001	$(1 * 2^3) + (0 * 2^2) + (0 * 2^1) + (1 * 2^0)$	9
1010	$(1 * 2^3) + (0 * 2^2) + (1 * 2^1) + (0 * 2^0)$	10
1011	$(1 * 2^3) + (0 * 2^2) + (1 * 2^1) + (1 * 2^0)$	11
1111	$(1 * 2^3) + (1 * 2^2) + (1 * 2^1) + (1 * 2^0)$	15

4 bits

KNOX GAME DESIGN

Binary - Addition

- 0 + 0 = 0
- 0 + 1 = 1
- 1 + 1 = 10 (0 and carry the 1)
- 1 + 1 + 1 = 11 (1 and carry the 1)

	0100	4
+	0111	7
=	1011	11

	1010	10
+	0101	5
=	1111	15

	1001	9
+	0111	7
=	0000	0 (4 bit overflow)

	0000 1001	9
+	0000 0111	7
=	0001 0000	16

Binary - Subtraction

- Binary two's complement: swap 0's to 1's, and 1's to 0's, then add 1
- Convert number to be subtracted to two's complement
- Add two numbers
- Remove leading 1

	1000	8
-	0101	5

Two's complement of 5 =
0101 => 1010 + 0001 => 1011

	0 1000	8
+	0 1011	5 complement
=	X 0011	3

	1110	14
-	0111	7

Two's complement of 7 =
0111 => 1000 + 0001 => 1001

	0 1110	14
+	0 1001	7 complement
=	X 0111	7

KNOX GAME DESIGN

Base 16 - Hexadecimal

1 digit (4-bit)
0
1
2
3
4
5
6
7
8
9
A
B
C
D
E
F

2 digits (8-bit)
00
01
02
09
0A
0B
0E
0F
10
11
1F
20
80
A0
FE
FF

Hexadecimal, Decimal, Binary

0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

1 hex digit (4-bit)

00	0	0000 0000
01	1	0000 0000
02	2	0000 0010
09	9	0000 1001
0A	10	0000 1010
0B	11	0000 1011
0E	14	0000 1110
0F	15	0000 1111
10	16	0001 0000
11	17	0001 0001
1F	31	0001 1111
20	32	0010 0000
80	128	1000 0000
A0	160	1010 0000
FE	254	1111 1110
FF	255	1111 1111

2 hex digits (8-bit)

KNOX GAME DESIGN